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Determination of Minimum Thrust Requirement for a Passenger Aircraft

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This paper describes a methodology aimed at a rapid assessment of the minimum thrust requirement for a given aircraft configuration. This latter involves a tight coupling between flight mission analyses, engine performance, and optimization techniques. The flight performance analysis is used to assess the relatively significant constraints, thus rapidly identifying the feasible design space to meet specific requirements, considering a tradeoff between operation constraints and objectives often of conflicting nature. With conventional procedures, it is rather difficult to find the optimum design point (match point), because of many involved objectives and constraints. However, by adopting the actual methodology using a subroutine called KSOPT, which solves constraints optimization problem using Kreisselmeier–Steinhauser envelope function formulation, associated with a deterministic optimization algorithm, either sequential quadratic programming or modified method of feasible direction, the overall procedure becomes simpler, because there is only one objective and a constraint, thus avoiding separate optimizations. This present methodology is therefore more effective from the point of view of rapidity in searching for the optimum match point, and may provide guidance in identifying a potential propulsion system and the necessary data serving to develop a derivative (growth) engine.

Nomenclature

C_D = drag coefficient
C_L = lift coefficient
D = drag force
$\boldsymbol{F}$ = vector of objective functions
g = constraints of inequality
L = lift force
M = Mach number
S = lifting surface
T = total thrust
V = velocity
W = weight
$\boldsymbol{X}$ = vector of variables

Subscripts

CA = climb in altitude
CR = cruise
ICS = initial climb segment
LA = landing
TO = takeoff
KS = envelope function

Introduction

IN EARLY preliminary (conceptual) design or in detail design, the study of feasible design options and the matching between a propulsion system and a configuration of an aircraft are of paramount importance, to arrive at a configuration that is not only feasible but also best satisfies a prescribed mission envelope, in terms of performance and cost-effectiveness. Typical parameters being optimized include wing loading, thrust-to-weight ratio, wing aerodynamics, and high-lift device design, thus leading to multi-objective optimization problems which have been receiving increased interest [1–3]. Roskam [4] and Raymer [5] have reviewed the conceptual design process for several aircraft configurations, starting from the request of proposals defining the flight mission profiles and the operating requirements in concordance with the Federal Air Regulation standards (FAR25). Other authors [6–8] have treated many aspects of flight performance by considering simplifying assumptions. When selecting and matching a propulsion system with a configuration of an aircraft, many possibilities may arise. Within the available power range of an aeroengine, the wing and high-lift device have to be designed in consequence by knowing the overall weight. On the other hand, for specified aircraft geometry, aerodynamics, and wing loading parameters, a candidate powering engine could be identified when minimizing the thrust-to-weight ratio relative to the feasible design domain.

The study of flight performance and constraints analyses as addressed in this paper served to generate the diagram of constraints, reveal the critical constraints and feasible design space, obtain the ideal match point by means of an optimization process, and subsequently identify the convenient propulsion system. The actual optimization procedure, based on the subroutine KSOPT, is implementing the envelope function KS associated with a deterministic algorithm using either the sequential quadratic programming SQP or the modified method of feasible direction MMFD. The motivation for such a preference is its simplicity and subsequent reduced number of constraints and objectives, thus leading to less complicated computations. Therefore, with this approach, it is unnecessary to plot all involved constraints by means of usual traditional methods, and then treat them separately. But, with this procedure, the optimum match point could be simply determined by a mono-objective optimization technique. This methodology was validated upon a subsonic twin-engine, long-range airliner Boeing B767-300, and resulted in the identification of the optimum match point comparable to that in practice, which helped to identify certain categories of engines that best satisfy the power requirement. These obtained results constitute the first part of an overall methodology being developed to assist the preliminary or detail design, which involves a tight coupling between optimization, flight mission analyses, and propulsion performance. Furthermore, it is mainly intended to reduce the computation steps and permits simpler interdisciplinary communication requirements.

Flight Missions and Constraints Analyses

The optimum match point between an engine and a configuration of an aircraft is mutually dependant on the available engine propulsive performance, flight mission profile (Fig. 1), and

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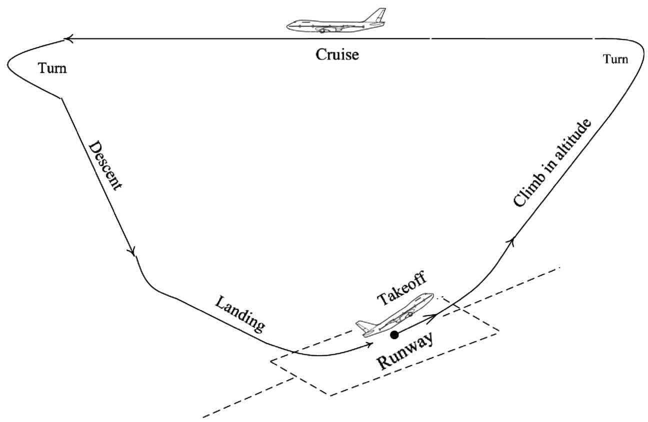

Fig. 1 Typical flight mission profile for a passenger airplane.

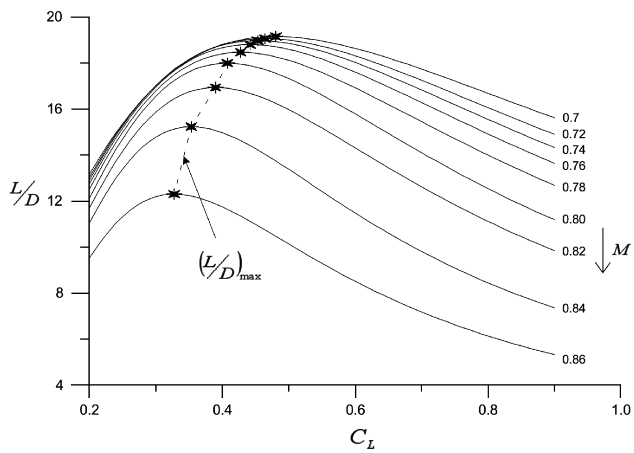

Fig. 2 Lift-to-drag ratio with Mach number.

constraints, which are often diverse and even extreme. Accordingly, an inhouse program was used to carry out such analyses, which includes aerodynamics and flight performance module coupled with another module for engine performance prediction. The basic data† (in addition to [9,10]) related to this model of commercial aircraft are summarized in Table 1. The aerodynamic polar (Fig. 2), as generated according to [11], is depicting a maximum lift-to-drag ratio that corresponds to an optimal flight velocity. The high-lift device used in this model of airplane, which incorporates a slat plus triple-slotted flap, permits reaching higher values of lift coefficient at takeoff and landing equal to 2.8 and 3.14, respectively [10,12]. The parameters of concern in such analyses are field lengths at takeoff and landing, climb rate/time, optimum cruising speed, and altitude which correspond to the maximum endurance and minimum fuel consumption. The segments of takeoff, climb, cruise, and landing are considered as the critical phases (Fig. 1), which are assessed separately in terms of thrust-to-weight ratio $(T/W)_{TO}$ (given in N/N) and wing loading $(W/S)_{TO}$ (given in N/m^2) referring to takeoff conditions. It should be noticed that high turn rates depending on the maximum lift coefficent and engine power are not considered herein. The takeoff constraint (line TO) as shown in Fig. 3 is mainly dependant on the wing loading, thrust-to-weight ratio, maximum lift coefficient, and takeoff distance which are related by an equation available in [4]. The climb in the takeoff segment occurs in three phases, in which the aircraft must demonstrate a performance no less than a specified climb gradient, considering a possible engine failure on takeoff. Accordingly, this constraint was assessed by a modified equation from [13], hence resulting in horizontal performance curves, where the critical phase is the initial climb segment (ISC)

†Data available online at the Boeing 767 family official web site, www.boeing.com/commercial/airports/acaps/767.pdf.

Table 1 Airplane characteristics† [9,10]

Characteristics	Values
Maximum weight at takeoff, kg	181,437
Ratio of weight at descent/takeoff	0.80
Ratio of fuel weight/takeoff	0.398
Maximum fuel capacity, liters	91,380
Range, km	9700–11,065
Typical cruise speed and Mach	851 km/h, 0.80
Cruise altitude, m	10,668
Wing lift surface, m^2	283.35
Span, m	47.60
Aspect ratio	7.99
Engines: two high by pass turbofan	——

Table 2 Values of thrust-to-weight ratio during climb in takeoff

	$(T/W)_{TO}(N/N)$
Initial climb segment ICS	0.2873
Transition segment of climb TSC	0.283
En route climb segment RCS	0.2663

(Table 2). The climb in altitude segment CA occurs at an angle and a rate of climb that correspond to maximum lift-to-drag ratio. The thrust-by-weight ratio, wing loading, and rate of climb are related by a similar equation as given in [4]. During the cruise flight segment (CR) the relationship between thrust-to-weight ratio and wing loading must satisfy the ideal cruise Mach number and altitude, while operating at a maximum lift-to-drag ratio and minimum fuel consumption. The aeroengines performance in terms of thrust and fuel consumption are basically function of altitude and airspeed, however, initial approximates [14] for the thrust ratios and weight were used, afterward adjusted by computations using the engine performance prediction module. The curves of landing, contrary to that of takeoff, are vertical and dependant solely on the value of lift coefficient at landing $(C_{L\,max})_{LA}$ for a given landing distance and specified approach angle and speed.

The subsequent constraint diagram (Fig. 3) was obtained by superimposing the critical constraint curves. The lines between the unacceptable sides of the constraints (with attached hatching) delimit the feasible design space (FS), in which the wing loading and thrust-to-weight ratio are the key parameters in identifying a potential propulsion system. The determined match point (design point) is

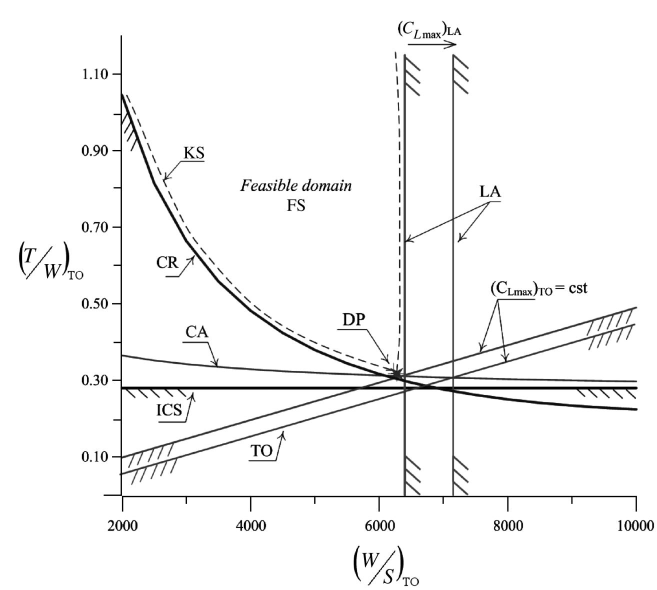

Fig. 3 Diagram of constraints for the model of airplane B767.

lying close to the intersection of the constraint curves, which is usually desirable for the purpose of reducing the engine size and wing geometry by minimizing the thrust-to-weight ratio and maximizing wing loading, respectively.

Optimum Match Point

The optimization procedure aiming at the determination of the best match point between the actual aircraft configuration and a propulsive system was implemented according to the flowchart shown in Fig. 4. As far as many constraint boundaries are involved, one has recourse to a multi-objective optimization, thus leading to a complicated computation process. However, this difficulty will be overcome by using the envelope function formulation, also known as the cumulative function KS (Kreisselmeier–Steinhauser) [15,16], which has been successfully used in various aircraft design applications [17–19] and has proved its ability in dealing with many multi-objective optimization problems [20,21]. Indeed, with this current formulation, we may avoid generating the constraint diagram by a traditional point-by-point method which is fairly tedious, particularly in cases of complex flight mission profiles (military fighters) involving many objectives to be satisfied simultaneously. In addition, with this actual methodology, there is no need to carry out separate optimizations as usually required in other methods, such as the Global Criterion Formulation or Penalty Function methods for example. The number of boundaries representing J-dimensional space is thus replaced with a single surface representative of all critical boundaries by using the cumulative function KS defined as follows:

$$\boldsymbol{F}_{\mathrm{KS}} = \frac{1}{\alpha} \ell n \left[\sum_{j=1}^{J} \exp(\alpha \boldsymbol{F}_j) \right] \tag{1}$$

When referring to the whole critical flight segments ($J = 5$), the KS function could be written as follows:

$$\boldsymbol{F}_{\mathrm{KS}} = \frac{1}{\alpha} \ell n \{ e^{\alpha (T/W)_{\mathrm{TO\_TO}}} + e^{\alpha (T/W)_{\mathrm{TO\_ICS}}} + e^{\alpha (T/W)_{\mathrm{TO\_CA}}} + e^{\alpha (T/W)_{\mathrm{TO\_CR}}} + e^{\alpha (T/W)_{\mathrm{TO\_LA}}} \}$$

The parameter α controls the distance of the KS function surface from the maximum values of boundary functions. Typical values of α range from 5 to 200 according to [22]. As long as a single objective function is now considered with one constraint, a nonlinear

Table 3 High bypass turbofans from different manufacturers

	Type of engine	Thrust at takeoff, lb	Thrust at takeoff, kN
RR	RB-211-524G	58,000	257.996
	RB-211-524H	60,600	269.562
GE	CF6 80 A2	50,000	222.411
	CF6 80 C2-B2	52,500	233.531
	CF6 80 C2-B4	57,900	257.551
	CF6 80 C2-B6	61,500	273.565
	CF6 80 C2-B7F1	60,600	269.562
	CF6 80 C2-B8	60,600	269.562
	CF6 80 E1-A2	67,500	300.255
P&W	JT9D-7R4E	50,000	222.411
	PW 4052	50,200	223.300
	JT9D-7Q	53,000	235.755
	PW 4056	56,750	252.436
	PW 4060	60,000	266.900
	PW 4062	60,600	269.562
	PW 4168	68,000	302.479

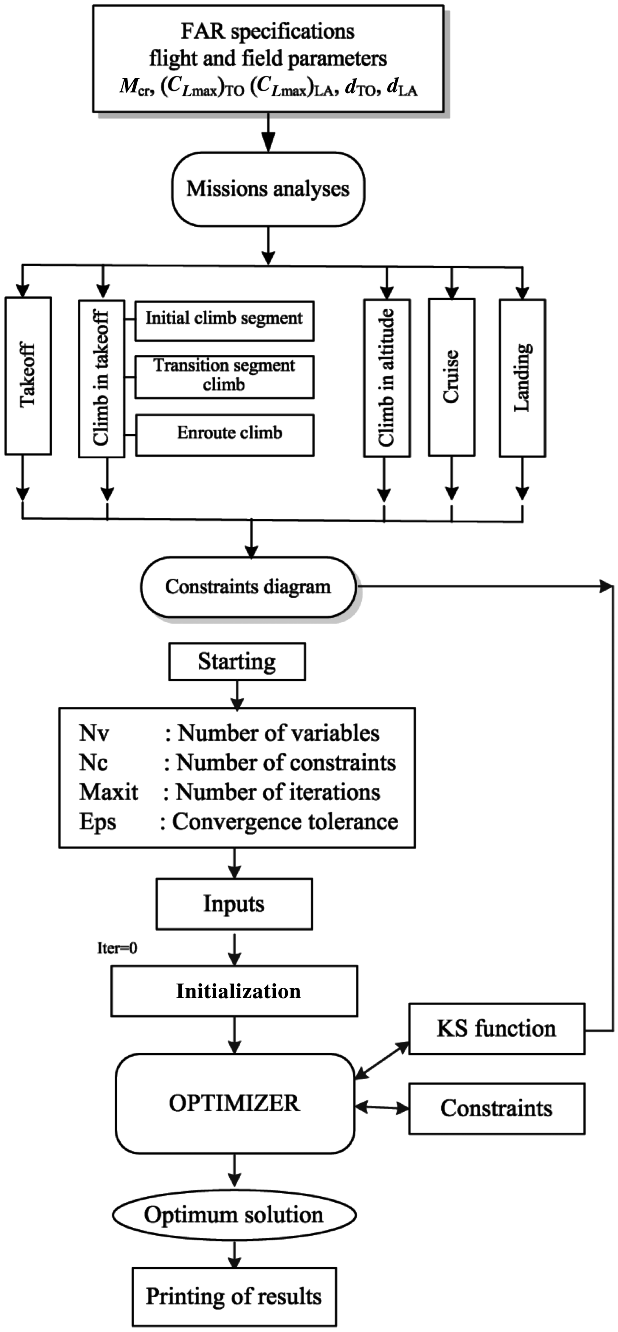

Fig. 4 Procedure for the determination of the optimum match point.

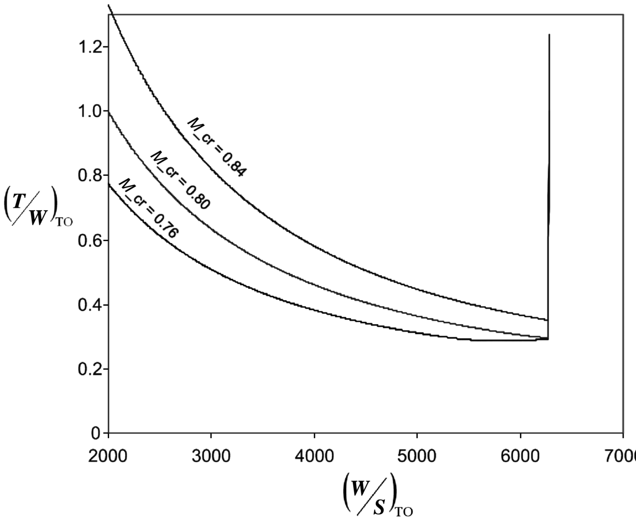

Fig. 5 Feasible domains with cruise Mach number.

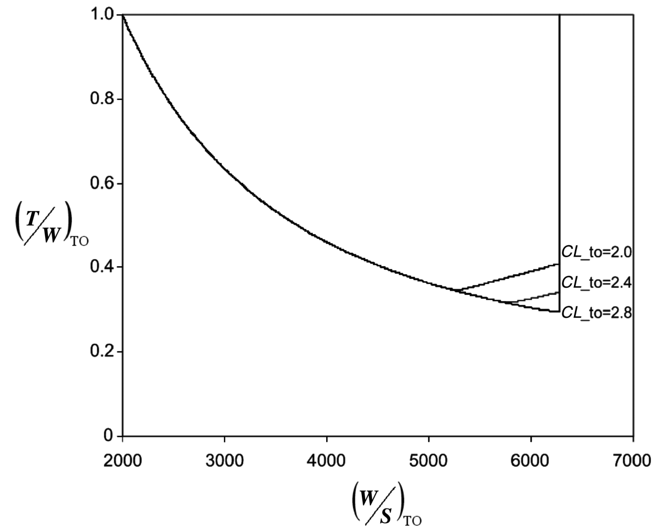

Fig. 6 Feasible domains with lift coefficient at takeoff.

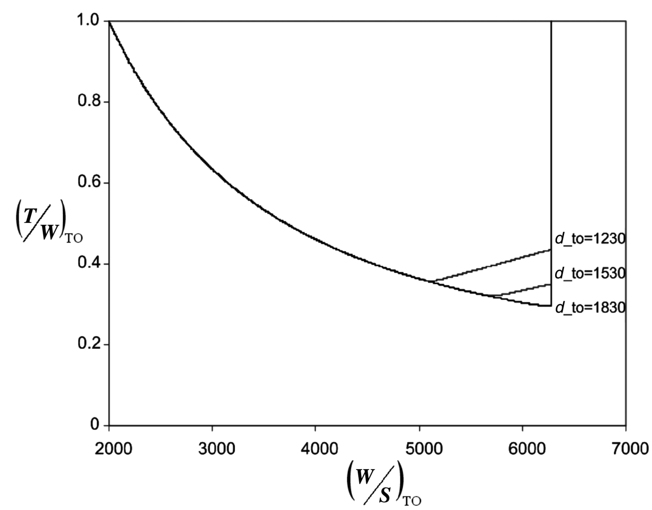

Fig. 7 Feasible domains with takeoff distance.

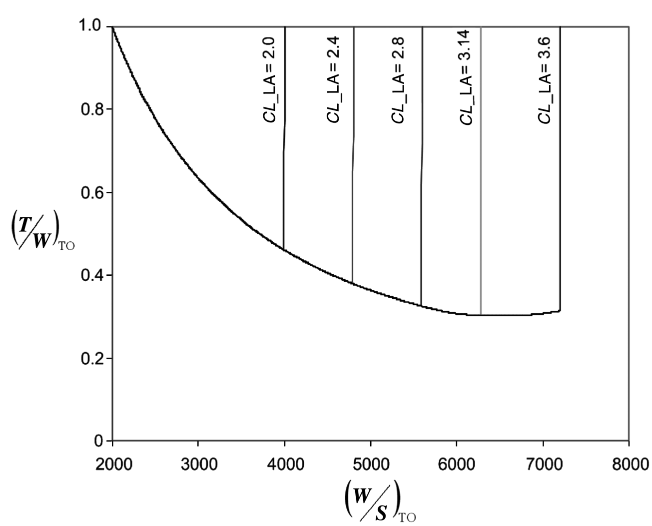

Fig. 8 Feasible domains with lift coefficient at landing.

deterministic algorithm, either MMFD or SQP, could be used to find the optimum. By considering the actual value of the wing loading $(W/S)_{\mathrm{TO}} = 6278.20\ \mathrm{N/m^2}$ as a unique constraint, this problem of optimization could be formulated as follows:

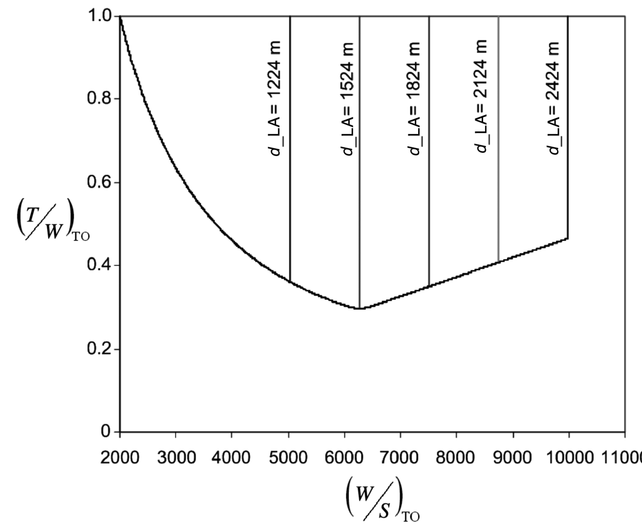

Fig. 9 Feasible domains with landing distance.

$$\begin{cases} \min[\boldsymbol{F}_{\mathrm{KS}}] \\ \text{subject to} \\ (W/S)_{\mathrm{TO,B767}} - (W/S)_{\mathrm{TO}} \le 0 \\ \text{and} \\ 2000 \le (W/S)_{\mathrm{TO}} \le 10,000 \end{cases} \tag{2}$$

Modified Method of Feasible Directions

The main idea of this method, initially developed by Zoutendijk [23], is to find a path from the initial feasible solution to the optimal solution by making steps along feasible directions. At each iteration step, the algorithm finds the optimal feasible direction and determines the step length that maximizes the objective function [24]. The critical part of the optimization task consists of finding of search direction S^q that minimizes the objective function while not violating any constraints. If the constraints are violated, S^q is given according to Zoutendijk [23], else it is calculated by the Fletcher and Reeves conjugate gradient method [25]. The optimization algorithm is written as follows:

$$\begin{array}{c} \min[\boldsymbol{F}(\boldsymbol{X})] \\ \text{subject to} \\ g_j(\boldsymbol{X}) \le 0 \qquad j = 1, \ldots, M \\ \boldsymbol{X}_i^L \le \boldsymbol{X}_i \le \boldsymbol{X}_i^U \qquad i = 1, \ldots, N \end{array} \tag{3}$$

The main steps are

1) Evaluate $\boldsymbol{F}(\boldsymbol{X}^{q-1})$ and $g_j(\boldsymbol{X}^{q-1})$ for $j = 1, M$.
2) Identify the set of critical constraints, J.
3) Calculate $\nabla \boldsymbol{F}(\boldsymbol{X}^{q-1})$ and $\nabla g_j(\boldsymbol{X}^{q-1})$.
4) Determine a search direction S^q.
5) Perform a one-dimensional search to find λ.
6) Set $\boldsymbol{X}^q = \boldsymbol{X}^{q-1} + \lambda S^q$.
7) Check for convergence.

Table 4 Optimum match points with cruise Mach number

Cruise Mach number	MMFD		SQP	
M_{cr}	$(W/S)_{\mathrm{TO}}$	$(T/W)_{\mathrm{TO}}$	$(W/S)_{\mathrm{TO}}$	$(T/W)_{\mathrm{TO}}$
0.76	6292.293	0.294339	6286.338	0.294118
0.80	6288.673	0.296631	6286.307	0.296615
0.82	6532.923	0.308501	6286.077	0.316246
0.84	6874.289	0.324676	6285.688	0.349928

Table 5 Optimum match points with lift coefficient at takeoff

Maximum lift coefficient at takeoff	MMFD		SQP	
$(CL_{\max})_{TO}$	$(W/S)_{TO}$	$(T/W)_{TO}$	$(W/S)_{TO}$	$(T/W)_{TO}$
2.0	6280.346	0.409758	6285.001	0.410062
2.2	6281.908	0.372600	6285.431	0.372809
2.4	6280.955	0.341499	6285.790	0.341761
2.6	6280.710	0.315267	6286.092	0.315533
2.8	6288.673	0.296633	6286.307	0.296615

Table 6 Optimum match points with takeoff distance

Takeoff distance	MMFD		SQP	
d_{TO}, m	$(W/S)_{TO}$	$(T/W)_{TO}$	$(W/S)_{TO}$	$(T/W)_{TO}$
930	6280.351	0.575928	6283.085	0.576179
1230	6281.300	0.435520	6284.705	0.435760
1530	6281.210	0.350120	6285.690	0.350372
1830	6288.673	0.296631	6286.307	0.296615

Sequential Quadratic Programming

This technique, initially developed by Schittkowski [26], is one of the most effective algorithms for solving constrained optimization problems, owing to its convergence. Basically, a Taylor series is used for the objective and constraint functions, then a quadratic approximate objective function with linearized constraints is used to create a direction finding of the form

$$\begin{aligned} &\min_{d\in R^n}\left\{\tfrac{1}{2}d^T B_k d + \nabla f(x_k)^T d\right\} \\ &\qquad\text{subject to} \\ &\nabla g_j(x_k)^T d + g_j(x_k) = 0 \qquad j = 1,\ldots,M_e \\ &\nabla g_j(x_k)^T d + g_j(x_k) \geq 0 \qquad j = M_e+1,\ldots,M \\ &x_l - x_k \leq d \leq x_u - x_k \end{aligned} \tag{4}$$

B_k is a positive definite approximation of the Hessian. A line search is used to find a new point ($x_{k+1} = x_k + \lambda d\,\lambda \in [0,1]$), such that a merit function will have a lower value at the new point. If optimality is not achieved, B_k is updated according to the modified Broyden–Fletcher–Goldfarb–Shanno (BFGS) formula [27]. More details about this technique are available in [26,28,29].

Results and Discussions

The diagram of constraints resulting from the superimposition of the critical flight constraints of the current aircraft configuration permits identifying the feasible design space as depicted by Fig. 3, which also illustrates that if a specific constraint is too demanding it overrides all others, hence reducing the allowable domain for design. The wing loading and the thrust-to-weight ratios are the key parameters helping to determine the best match point and subsequently identify a candidate engine, or provide a preliminary requisite for improving engine power (case of growth engine). The final selection of a particular pair of values for an efficient matched design (DP) usually dictates to decide on the engine design itself, and even to modify the overall design for an aircraft. Definitely, for a maximum weight at takeoff, the engine power is fixed by the value of thrust-to-weight ratio, whereas the gross wing area is fixed by the value of wing loading. Furthermore, the minimum thrust-to-weight ratio for an imposed wing loading (known aircraft weight) means a minimum size for the engine.

The envelope function formulation, as implemented for the prospect of reducing the number of constraints, was able to follow closely the feasible design space as depicted by Fig. 3. The optimum match point found by means of the deterministic algorithm SQP or MMFD was near the intersection of constraint lines, which is desirable and referred to as a well-balanced design. For a value of wing loading equal to 6278.20 N/m^2 and flight Mach number equal to 0.8, takeoff and landing distances equal to 1830 and 1524 m, respectively, in addition to lift coefficient at takeoff $(C_{L\max})_{TO} = 2.8$ and that of landing $(C_{L\max})_{LA} = 3.14$, the optimum match point was found to correspond to a thrust-to-weight ratio equal to 0.2966. Consequently, for the known total weight at takeoff, the required total installed (net) thrust at takeoff is equal to 538.032 kN, thus the unitary engine gross thrust is equal to 277.336 kN including installation drag. Within the thrust range of available aeroengines powering present-day commercial fleet (Table 3), there are several potential engines, however, the highlighted ones (in bold) seem to be more appropriate for that requirement.

To demonstrate further capabilities of the actual optimization methodology in case of varying operating conditions, a parametric study was carried out for cruise Mach number, lift coefficients at takeoff and landing, as well as field characteristics. The utilization of the cumulative function KS led to different trends for the feasible design domain, as depicted by Figs. 5–9, and the subsequent match points as presented in Tables 4–8. When referring to the effect of cruise Mach number (Fig. 5 and Table 4), it is obvious that more thrust is required at takeoff to reach higher cruise velocities. For example, at Mach number equal to 0.84, the required gross thrust at takeoff is equal to 303.588 kN (MMFD) and 327.174 kN (SQP) instead of its initial value of 277.336 kN. Concerning the effect of the maximum lift coefficient at takeoff (Fig. 6 and Table 5), it is clear that a more powerful engine is required if the aerodynamic lift coefficient is lower. For example, at $(C_{L\max})_{TO} = 2.0$ the gross thrust required at takeoff is set at 383.371 kN instead of the value of 277.336 kN. Similar behavior occurs with the takeoff distance (Fig. 7), hence, for a shorter distance of 1230 m, for example, the required gross thrust becomes 407.458 kN according to Table 6. The maximum lift coefficient at landing does not affect the engine power, but rather the maximum tolerated wing loading. In this instance, for a value of the coefficient $(C_{L\max})_{LA}$ less than 3.14, the field length must be extended; in contrary, this distance could be shortened (see Fig. 8 and Table 7). Similar effects are noticed (as seen from Fig. 9 and Table 8) when increasing the landing distance which is inversely proportional to the lift coefficient at landing $(C_{L\max})_{LA}$ for a given wing loading. These results demonstrate how the actual methodology can be used to rapidly verify whether the selected propulsion system could fulfill

Table 7 Optimum match points with lift coefficient at landing

Maximum lift coefficient at landing	MMFD		SQP	
$(CL_{\max})_{LA}$	$(W/S)_{TO}$	$(T/W)_{TO}$	$(W/S)_{TO}$	$(T/W)_{TO}$
2.8	——	——	——	——
3.14	6288.673	0.296631	6286.307	0.296615
3.4	6278.194	0.296574	6278.194	0.296574
3.6	6278.194	0.296574	6278.194	0.296574

Table 8 Optimum match points with landing distance

Takeoff distance	MMFD		SQP	
d_{LA}, m	$(W/S)_{TO}$	$(T/W)_{TO}$	$(W/S)_{TO}$	$(T/W)_{TO}$
1224	6278.194	0.296574	6278.194	0.296574
1524	6288.673	0.296631	6286.307	0.296615
1824	6278.194	0.296574	6278.194	0.296574
2124	6278.194	0.296574	6278.194	0.296574

all possible extreme operating conditions. Finally, when comparing between the used algorithms SQP and MMFD, and in view of the experienced computations, the first algorithm seems to provide more reasonable results and better convergence and CPU time in such design optimization problems.

Conclusions

This paper presents a methodology aimed at the determination of the optimum match point in term of minimum thrust requirement for a given aircraft configuration. The flight performance and constraints analyses resulted in identifying the design feasible space bounded by the cumulative function KS. The envelope function formulation was preferred, owing to its great advantage in reducing the number of variables and constraints, hence simplifying the overall optimization procedure. The obtained optimum match point is shown to correspond to a well-balanced design in terms of wing loading and thrust-to-weight ratio, which could provide guidance either in identifying a candidate engine or in optimizing the design for a derivative engine. This methodology, as validated upon an existing model of B767-300, is thus concluded to be viable in such kind of multi-objectives optimization problem, from the point of view of effectiveness and rapidity when associated with robust deterministic algorithms. Also, it can be used to analyze extreme operating conditions, match, and develop optimized derivative engines.

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